New trends in arithmetic combinatorics and related fields

Schedule and Abstracts

Institute of Mathematics at the University of Granada (IMAG)
1 – 6 June 2025

Schedule

Time	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
09:00 – 09:30 09:15 – 09:30	*Registration* *IMAG welcome*				
09:30 – 10:15	Bloom	Prendiville	Van Hintum	Kuca	Bedert
10:15 – 11:00	Pach	Iosevich	Matolcsi	Szegedy	Matthiesen
11:00 – 11:30	*Coffee*	*Coffee*	*Coffee*	*Coffee*	*Coffee*
11:30 – 12:15	Milićević	Roche-Newton	Tointon	Leng	Thompson
12:15 – 13:00	Elsholtz	Petridis	Grynkiewicz	Altman	Teräväinen
13:00 – 15:00	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
15:00 – 15:45	Shkredov	Chapman	Free afternoon Granada visits (Alhambra)	Pilatte	*End*
15:45 – 16:30	Wolf	Solymosi		Dartyge	
16:30 – 17:00	*Coffee*	*Coffee*		*Coffee*	
17:00 – 17:45	Campos	Rudnev		Montejano	
17:45 – 18:30	Peluse	*Problem session*		*Evening math session*	
19:30 – 21:30	*Dinner*	*Dinner*		*Dinner*	
20:30			*Conference dinner*		

All the talks take place in the main conference room at the IMAG.

The meals indicated in the schedule are offered at Hotel Turia, except the conference dinner. Breakfast is also offered at Hotel Turia, from 7:00 to 10:00.

The conference dinner takes place in *Carmen de la Victoria*, Cta. del Chapiz, 9, Albaicín, 18010 Granada.

Speakers

Abstracts

Transference in the quantitative polynomial Szemerédi theorem

Daniel Altman, *Standford University*

We will discuss a framework which uniformly recovers and extends all known cases of quantitative bounds in the finite field polynomial Szemerédi theorem. We will also discuss its usage in the integer setting, showing in particular quantitative bounds for Szemerédi's theorem with (fixed) polynomial common difference. Based on joint work with Mehtaab Sawhney.

Large Sum-free sets via L^1-estimates for trigonometric series

Benjamin Bedert, *University of Oxford*

A set B is said to be *sum-free* if there are no $x, y, z \in B$ with $x + y = z$. A classical probabilistic argument of Erdős shows that any set of N integers contains a sum-free subset of size $N/3$, and this was later improved to $(N+2)/3$ by Bourgain using a Fourier-analytic approach. We show that there exists a constant $c > 0$ such that any set of N integers contains a sum-free subset of size $N/3 + c \log\log N$, confirming a longstanding suspicion in additive combinatorics. A key step in the proof consists of establishing inverse results giving combinatorial descriptions for sets of integers whose Fourier transform has small L^1-norm.

Control in additive combinatorics and its applications

Thomas Bloom, *University of Manchester*

Bounds for the third moment of the convolution have played an important role recently in additive combinatorics, most notably in the study of the sum-product problem and the growth of convex sets of real numbers. In this talk I will give a unified overview of how and why such third moment bounds are useful in additive combinatorics, including recent advances in the sum-product problem, growth of convex sets, and the Balog-Szemerédi-Gowers theorem.

On the independence number of Sparser Cayley Graphs

Marcelo Campos, *IMPA*

Given a p-random set $A \subseteq \mathbb{Z}_n$, the random Cayley graph Γ_p is defined to have vertex set $\mathbb{Z}_n$ and an edge between two distinct vertices $x, y \in \mathbb{Z}_n$ if $x + y \in A$. For $p = 1/2$ Green and Morris showed that the independence number of $\Gamma_{1/2}$ is asymptotically equal to $\alpha(G(n, 1/2))$. In this talk I'll show that the independence number of Γ_p matches that of $G(n,p)$ for $p \geq (\log n)^{-1/80}$.

This is joint work with Gabriel Dahia and João Pedro Marciano.

Additive Ramsey theory over Piatetski-Shapiro numbers

Jonathan Chapman, *University of Warwick*

A foundational theorem of Schur shows that any finite colouring of the positive integers produces x, y, z all with the same colour such that $x + y = z$. A famous open-problem of Erdős and Graham asks whether this result remains true if we require the monochromatic x, y, z to all be squares. In this talk, we will investigate the more general problem of colouring *Piatetski-Shapiro sequences* $\{\lfloor n^c \rfloor : n \in \mathbb{N}\}$, for some fixed real number $1 \leqslant c \leqslant 2$. By combining ideas from additive combinatorics and harmonic analysis, we prove that the analogue of Schur's theorem holds for all Piatetski-Shapiro sequences with $c < 12/11$. We also establish more general Ramsey-theoretic results concerning solutions to linear equations over monochromatic or relatively dense subsets of Piatetski-Shapiro sequences.

This talk is based on joint work with Sam Chow (University of Warwick) and Philip Holdridge (University of Warwick).

The reverses of the primes and of the almost primes

Cécile Dartyge, *Institut Élie Cartan de Lorraine*

Let b be an integer bigger than 2. The reverse in base b of an integer $a = \sum_{j=0}^{n-1} e_j(a) b^j$ with $e_j(a) \in \{0, \ldots, b-1\}$ for $j = 0, \ldots, n-1$ is given by $\overleftarrow{a} = \sum_{j=0}^{n-1} e_j(a) b^{n-j-1}$. For example in base $b = 10$ we have $\overleftarrow{13} = 31$. In this talk, I will present several recent results on the reverse of the primes and the almost prime numbers. This talk is based on a joint work with Bruno Martin, Joël Rivat, Igor E. Shparlinski and Cathy Swaenepoel and a second work in progress with Joël Rivat and Cathy Swaenepoel.

Sumsets in the set of squares and progression-free sets

Christian Elsholtz, *Graz University of Technology*

I will first give a survey on previous results on sumsets in the set of integer squares, (e.g. work by Gyarmati, Hegyvari and Sárközy, and Dietmann, Shparlinski and myself).

I will then report on recent work with Lena Wurzinger. For example we show: If $A+B \subset [1, N]$ is a subset of the squares, and $|A| \geq c \log \log N$, then $|B| = O(\log N \log \log N)$.

Then, I will recall various well known connections of sumsets and arithmetic progressions. In joint work with Imre Ruzsa and Lena Wurzinger we present a new connection: Let G be a commutative group, in which every nonzero element has order at least $2k+1$. Let $A, B \subset G$ be sets such that $A+B = \{a+b : a \in A, b \in B\}$ does not contain an arithmetic progression of length $2k+1$. Then one achieves a nontrivial sumset growth $|A+B| \geq |A|^{\alpha_k}|B|^{\beta_k}$. This also has consequences for ternary sumsets in the set of integer squares.

Inverse Structure for High Density Sumsets $A+B$ Modulo a Prime

David J. Grynkiewicz, *University of Memphis*

The $3k-4$ Theorem for $\mathbb{Z}$ asserts that, if A, $B \subseteq \mathbb{Z}$ are finite, nonempty subsets with $|A| \geq |B|$ and $|A+B| = |A|+|B|+r < (|A|+|B|-3)+|B|$, then there exist arithmetic progressions P_A and P_B of common difference such that $X \subseteq P_X$ and $|P_X| \leq |X|+r+1$ for all $X \in \{A, B\}$. There are numerous partial versions of the $3k-4$ Theorem modulo a prime p. The focus of this talk is a recent extension valid for *general* sumsets $A+B$ at *high density*, both of which increase the difficulty. The ideal conjectured density restriction under which such a version of the $3k-4$ Theorem modulo p is expected is $|A+B| \leq p-(r+3)$, which includes sumsets $A+B$ with density $1-\epsilon$. Under this ideal density constraint, there are arithmetic progressions P_A, P_B and P_C of common difference with $X \subseteq P_X$ and $|P_X| \leq |X|+r+1$ for all $X \in \{A, B, C\}$, where $C = -G \setminus (A+B)$, provided $|A+B| = |A|+|B|+r \leq (|A|+|B|-3)+0.01|B|$. This generalizes a result of Serra and Zémor by extending their work from the special case $A+A$ to that of general sumsets $A+B$, removes unnecessary sufficiently large p restrictions, and improves (also in the case $A=B$) their constant 100-fold, from 0.0001 to 0.01. Moreover, at the cost of a near optimal density restriction of optimal order of magnitude $|A+B| \leq p-O(r)$, a yet better 1000-fold improvement in constants is possible. The proof combines finite fourier analysis, isoperimetric methods, and combinatorial reduction arguments, mixing new ideas with variations of methods used previously by Serra and Zémor, Lev and Shkredov, Freiman, Candela and González-Sánchez, Huicochea, and others.

Exact signal recovery times series imputation

Alex Iosevich, *University of Rochester*

We are going to describe a frequently arising practical problem where a positive percentage of a time series is unavailable and needs to be recovered before a forecasting process can take place. We are going to see that imputing missing values can be accomplished very efficiently using ideas from the theory of exact signal recovery. These ideas are, in turn, based on fundamental results due to Bourgain, Talagrand, and others. The talk is based on joint work with Azita Mayeli, and another joint work with Burstein, Mayeli and Nathan.

Quantitative concatenation and polynomial corners

Borys Kuca, *Jagiellonian University*

The polynomial Szemerédi theorem of Bergelson and Leibman is one of the most important generalizations of the celebrated theorem of Szemerédi on arithmetic progressions. Originally proved using ergodic-theoretic methods, it lacks a quantitative version to this day. Recent years have seen dramatic progress in obtaining bounds for sets avoiding various classes of polynomial configurations. In this talk, I will discuss one such result, joint with Noah Kravitz and James Leng, together with various technical tools that we have developed on the way.

Szemerédi's theorem, primes, and nilsequences

James Leng, *University of California, Los Angeles, UCLA*

Let $r_k(N)$ be the largest subset of $[N] = \{1, \ldots, N\}$ with no k-term arithmetic progression. Szemerédi's theorem states that $r_k(N) = o_k(N)$. We will go over the proof that achieves the best known upper bounds for $r_k(N)$ for general k. We will discuss how the mathematics behind the proof relates to counting primes along linear forms and the distribution of orbits on G/Γ with G nilpotent and Γ discrete and cocompact. This is (partly) based on joint work with Ashwin Sah and Mehtaab Sawhney.

Functional tilings and the Coven-Meyerowitz conditions

Máté Matolcsi, *HUN-REN Alfréd Rényi Institute of Mathematics*

Coven and Meyerowitz formulated two conditions, (T1) and (T2), which have been conjectured to characterize all finite sets that tile the integers by translation. By periodicity, this conjecture can equivalently be formulated for tilings of finite cyclic groups $A \oplus B = \mathbb{Z}_M$. In this paper we consider a natural relaxation of this problem, where we replace sets with nonnegative functions f, g, such that $f(0) = g(0) = 1$, $f * g = \mathbf{1}_{\mathbb{Z}_M}$ is a functional tiling, and f, g satisfy certain further natural properties associated with tilings. We show that the Coven-Meyerowitz tiling conditions do not necessarily hold in such generality. Such examples of functional tilings carry the potential to lead to proper tiling counterexamples to the Coven-Meyerowitz conjecture in the future.

Joint work with Gergely Kiss, Itay Londner and Gábor Somlai.

U^k-norms of smooth numbers

Lilian Matthiesen, *Universität Göttingen*

An integer is called y-smooth if all of its prime factors are of size at most y. We will report on work in progress on the problem of bounding the U^k-norms of the set of y-smooth numbers below N. These sets are sparse as soon as y is sufficiently small, that is if $y = N^{o(1)}$, and yet they enjoy very good distributional properties. From a technical perspective, this problem is an interesting new test case for known techniques of studying U^k-norms, since, in particular, standard constructions for pseudo-random majorants are no longer available once y is sufficiently small.

Freiman Bihomomorphisms in Finite Abelian Groups

Luka Milićević, *Mathematical Institute of the Serbian Academy of Sciences and Arts*

Given a set $A \subseteq G_1 \times G_2$ for some finite abelian groups G_1, G_2 and H, a map $\phi : A \to H$ is said to be a *Freiman bihomomorphism* if it respects all horizontal and vertical additive quadruples. In the case of finite vector spaces, structural results for such maps play a role in the proof of the quantitative inverse theorems for the Gowers U^4 norm.

In this talk, I will discuss a work in progress, which concerns the structure of such maps in finite abelian groups. I will also discuss applications of that theory to the inverse theorems for the U^4 norm in two settings; in finite abelian groups of order coprime to 6, and in groups where torsion is 2^k for a fixed k.

Discrete Brunn–Minkowski Inequalities: Convexity, dimension, and structure

Amanda Montejano, *Universidad Nacional Autónoma de México (UNAM)*

In recent years, there has been growing interest in discrete versions of the Brunn–Minkowski inequality. Although several versions have been proposed, there is still no clear consensus on what the "best" discrete formulation should be. In the continuous setting, a refinement of the Brunn-Minkowski inequality due to Bonnesen incorporates the $(d-1)$-dimensional volume of the projection of the sets onto a given hyperplane. This result has a discrete counterpart only in dimension two, due to Grynkiewicz and Serra. In this talk, we explore extensions and generalizations of that result to higher dimensions. In particular, we discuss a framework for deriving meaningful inequalities in arbitrary dimensions that also reflect the degree of convexity of the involved sets. Joint work with Luis Montejano and Oriol Serra.

The Alon-Jaeger-Tarsi conjecture via group ring identities

Péter Pál Pach, *Rényi Institute and BME*

The Alon-Jaeger-Tarsi conjecture states that for any finite field $\mathbb{F}$ of size at least 4 and any nonsingular matrix A over $\mathbb{F}$ there exists a vector x such that neither x nor Ax has a 0 component. In this talk we discuss the proof of this result for $|\mathbb{F}| > 79$ and further applications of our method about coset covers and additive bases. Joint work with János Nagy and István Tomon.

Additive combinatorics and pointwise ergodic theory

Sarah Peluse, *Stanford University*

Objects called "nonconventional ergodic averages" appeared for the first time in Furstenberg's proof of Szemerédi's theorem, and understanding the limiting behavior of these averages became an important problem in ergodic theory. After breakthrough work of Bourgain in the late 1980s and early 1990s, no further progress had been made on proving pointwise almost everywhere convergence of nonconventional ergodic averages until very recently. I will report on this progress, along with some of the key inputs from additive combinatorics.

Some questions on covering sets

Giorgis Petridis, *University of Georgia (USA)*

It is often useful to cover a set by translates of a "better understood" set. In this talk some known results will be surveyed and some open questions will be discussed.

Factoring in time $O(n^{3/2})$

Cédric Pilatte, *University of Oxford*

The security of widely used communication systems depends on the presumed hardness of factoring large integers or computing discrete logarithms. Shor's celebrated 1994 algorithm showed that quantum computers can solve these problems in polynomial time. In 2023, Regev proposed an even faster quantum algorithm for factoring integers, though its correctness relied on an unproven, ad hoc number-theoretic conjecture. In this work, we provide a provably correct version of Regev's algorithm by establishing a related result on the growth of product sets in $(\mathbb{Z}/N\mathbb{Z})^\times$. Specifically, for a subset $A \subset (\mathbb{Z}/N\mathbb{Z})^\times$, let $A^{(k)}$ denote the set of all products of at most k elements from A. We estimate the least $k \geq 1$ such that $A^{(k)}$ equals the subgroup generated by A, in the setting where A consists of approximately $\sqrt{\log N}$ randomly chosen primes in the interval $[1, e^{\sqrt{\log N}}]$. Our proof combines lattice-based methods with tools from analytic number theory.

An inverse theorem for the Gowers U^3-norm relative to quadratic level sets

Sean Prendiville, *Lancaster University*

The Gowers uniformity norms have become well-used tools in additive combinatorics, ergodic theory and analytic number theory. We discuss an effective version of the inverse theorem for the Gowers U^3-norm for functions supported on high-rank quadratic level sets in finite vector spaces. This enables one to run density increment arguments with respect to quadratic level sets, which are analogues of Bohr sets in the context of quadratic Fourier analysis on finite vector spaces. For instance, one can derive a polyexponential bound on the Ramsey number of three-term progressions which are the same colour as their common difference ("Brauer quadruples"), a result it seems difficult to obtain by other means.

Additive properties of convex sets

Oliver Roche-Newton, *Johannes Kepler Universität*

A finite set $A \subset \mathbb{R}$ is said to be *convex* if its consecutive differences are strictly increasing. That is, labelling the elements of A so that $a_1 < a_2 < \dots a_n$, we have that

$$a_i - a_{i-1} < a_{i+1} - a_i$$

holds for all $2 \leq i \leq n-1$. One expects that convex sets cannot be too additively structured, and there are various different problems which give different ways to quantity this belief. Perhaps the most well-known such problem is the conjecture of Erdős which states that the sum set of a convex set must have cardinality close to the maximum possible size $c|A|^2$.

In this talk (based on work in progress with Thomas Bloom and Jakob Führer), I will discuss some other additive questions concerning convex sets. The central question of the talk is the following: how many three-term arithmetic progressions can a convex set have? Some partial answers to this and closely related problems will be given.

On distinct angles in the plane

Misha Rudnev, *University of Bristol*

My attention to this question was drawn by Sergey Konyagin. What is the minimum number of distinct angles PQR defined by a non-collinear plane set of N points, where P, Q, R are points in the set? The problem is that you can have all the points on a circle plus possibly the centre or on a line plus one or two points outside the line, and have roughly N angles only. However, but for pruning these two examples, it appears that the number of distinct angles should be much larger. Say, is it true that if no three points lie on a line and no four on a circle, the number of distinct angles is $\Omega(N^2)$?

I am not aware of *any* nontrivial lower bound for the latter general position arrangement. But in the two special cases of (i) the point set respecting order or (ii) being a Cartesian product it is possible to prove lower bounds in the form $\Omega(N^{1+c})$ using fairly different techniques. The former case is a joint paper with Konyagin and Passant, and the latter was answered by Roche-Newton.

Some applications of the higher energy method to distribution irregularities

Ilya Shkredov, *Purdue University and MPIM*

We review recent results obtained by the method of higher sumsets and higher energies. In particular, we discuss two applications: irregularities in the distribution of the difference set and irregularities in the large Fourier coefficients of sets with small sumsets.

Elekes-Szabó type theorems and their connections to additive combinatorics

Jozsef Solymosi, *University of British Columbia*

A theorem of Elekes and Szabó recognizes additive or multiplicative substructures among certain complex algebraic varieties with large-size intersections with finite grids in three dimensions. This theorem is a central tool in proving results in discrete geometry and combinatorics, including some problems in additive combinatorics. I will briefly review the history of the theorem and its connections to other fields of mathematics. I will mention recent results and many open problems.

Spectral Algorithms in Higher-Order Fourier Analysis

Balázs Szegedy, *HUN-REN Alfréd Rényi of Mathematics*

The Fast Fourier Transform is arguably one of the most successful algorithms in applied mathematics. The emerging field of higher-order Fourier analysis raises a natural question: can it also be applied in practice? Are there higher-order analogues of decomposing functions into harmonic components? Can such decompositions be computed efficiently? And what are their potential applications? In this talk, we announce new progress on these questions. We show that appropriate operator representations of functions on abelian groups lead to a novel spectral framework for higher-order Fourier analysis—one with both theoretical significance and practical potential. Within this framework, we present spectral versions of arithmetic regularity lemmas, higher-order Fourier decompositions, and related algorithmic tools. Joint work with Pablo Candela and Diego Gonzalez-Sanchez

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Linnik's problem for the Möbius function

Joni Teräväinen, *University of Cambridge*

We show that the least natural number having an odd number of prime factors and belonging to any arithmetic progression $a \pmod{q}$ is bounded by $q^{2+o(1)}$. This can be seen as a multiplicative analogue of Linnik's problem on the least prime in an arithmetic progression. This is joint work with Kaisa Matomäki.

On a conjecture of Erdős, Granville, Pomerance, and Spiro

Lola Thompson, *Universiteit Utrecht*

Let $s(n)$ denote the sum of proper divisors of an integer n. The function $s(n)$ has been studied for thousands of years, due to its connection with the perfect numbers. In 1992, Erdős, Granville, Pomerance, and Spiro (EGPS) conjectured that if $\mathcal{A}$ is a set of integers with asymptotic density zero then $s^{-1}(\mathcal{A})$ also has asymptotic density zero. This has been confirmed for certain specific sets $\mathcal{A}$, but remains open in general. In this talk, we will give a survey of recent progress towards the EGPS conjecture. This talk is based on joint work with various subsets of the following co-authors: Kübra Benli, Giulia Cesana, Cécile Dartyge, Charlotte Dombrowsky, Paul Pollack, and Carl Pomerance.

Structure, expansion and probability in vertex-transitive graphs

Matthew Tointon, *University of Bristol*

Celebrated theorems of Gromov, Trofimov and Coulhon—Saloff-Coste combine to give a remarkable dichotomy for vertex-transitive graphs: such graphs must either resemble highly structured Cayley graphs, or must exhibit "expansion" in a certain sense. This in turn has had a number of striking applications, particularly to probability, such as Varopoulos's famous characterisation of those transitive graphs on which the random walk is recurrent (i.e. eventually returns to its starting point with probability 1). I will describe a number recent quantitative, finitary refinements of these results that allow us to give meaningful extensions of results like Varopoulos's to finite transitive graphs and finite regions of infinite transitive graphs.

Sharp Stability of the Prékopa-Leindler inequality

Peter van Hintum, *Institute for Advanced Study, Princeton*

The Prékopa-Leindler (PL) inequality serves as a functional analogue of the Brunn-Minkowski (BM) inequality on the size of sumsets in $\mathbb{R}^n$. As PL is obtained from BM as the number of dimensions tends to infinity, it can provide dimensionally independent results in the context of geometry, analysis, and probability, motivating the significant attention its stability has garnered in recent years. While a lot of progress has been made in the geometric setting of BM, a sharp quantitative stability result for the Prékopa-Leindler inequality has remained elusive. In this talk, we will explore these inequalities and present recent results that resolve the long-standing question of their quantitative stability.

Based on joint work with Alessio Figalli and Marius Tiba.

The structure of sets of bounded VC_2-dimension

Julia Wolf, *University of Cambridge*

In joint work with Caroline Terry, we showed that subsets of bounded VC_2-dimension in a high-dimensional vector space of a fixed finite field can be approximated by a union of atoms of a high-rank, bounded-complexity quadratic factor. This generalised prior work of Alon-Fox-Zhao, Sisask, and Conant-Pillay-Terry for subsets of bounded VC-dimension, and is analogous to joint work with Terry and qualitative results of Chernikov-Towsner in the setting of hypergraphs. In this talk we give a new perspective on the proof and explore the quantitative aspects of the argument.

Organizers:

Pablo Candela, *Instituto de Ciencias Matemáticas, ICMAT*

Diego González-Sánchez, *HUN-REN Alfréd Rényi Institute of Mathematics*

Harald Helfgott, *CNRS, Institut de Mathématiques de Jussieu (Paris VI/VII)*

Anne de Roton, *Institut Élie Cartan de Lorraine*

Alisa Sedunova, *Purdue University*

Oriol Serra, *Universitat Politècnica de Catalunya*